

Learning Goals

1. Understand the meaning of necessary and sufficient conditions (carried over from Wednesday).
2. Understand the difference between a valid argument and an invalid argument.
3. Be able to use a truth table to determine the validity of an argument.
4. Be able to use rules of inference to determine the validity of an argument.
5. Apply the process of looking for truth values that would demonstrate that an argument is invalid.

Issues raised in Question 5 of the quiz:

Using the rules of inference.

Trying to find truth values that make all premises true but conclusion false.

The textbook says a sound argument is a valid argument with true premises. Can you have a valid argument with premises that are not true?

Can you prove Q3 from the quiz using rules of inference?

Can you explain Q4 from the quiz?

Necessary and sufficient conditions (carried over from Wednesday):**Activity 1:**

Let n be a positive integer.

(a) State a condition that is necessary, but not sufficient, for n to be divisible by 6.

(b) State a condition that is sufficient, but not necessary, for n to be divisible by 6.

(c) State a necessary and sufficient condition for n to be divisible by 6.

Argument form:**Activity 2:**

Consider the following argument:

*If wages are raised, buying increases. If there is a depression, buying does not increase.
Therefore there is not a depression or wages are not raised.*

Let w represent the statement “wages are raised”,
 b represent the statement “buying increases”, and
 d represent the statement “there is a depression”.

Write this argument in symbolic form.

Premise 1: $w \rightarrow b$

Premise 2: $d \rightarrow \sim b$

$$((w \rightarrow b) \wedge (d \rightarrow \sim b)) \rightarrow (\sim d \vee \sim w)$$

Conclusion: $\sim d \vee \sim w$

Proving an argument is valid (truth table method):**Premise 1:** $w \rightarrow b$ **Premise 2:** $d \rightarrow \sim b$ **Conclusion:** $\sim d \vee \sim w$

w	b	d	$\sim w$	$\sim b$	$\sim d$	$w \rightarrow b$	$d \rightarrow \sim b$	$\sim d \vee \sim w$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	F	T	F
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	T	F	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

**Question 1 (Assessed):** This argument is:

- A. valid.
- B. Invalid.

Proving an argument is valid (rules of inference method):**Premise 1:** $w \rightarrow b$ **Premise 2:** $d \rightarrow \sim b$ **Conclusion:** $\sim d \vee \sim w$ **Activity 4:** Use the laws of logical equivalence and the rules of inference to show that this argument is valid.**1:** $w \rightarrow b$ **2:** $d \rightarrow \sim b$ **3:** $b \rightarrow \sim d$ from 2 by taking the contrapositive**4:** $w \rightarrow \sim d$ from 1 and 3 by the Transitivity argument form**5:** $\sim w \vee \sim d$ from 4 by rewriting \rightarrow **Conclusion:** $\sim d \vee \sim w$ from 5 by commutative law

Q3 from the L5 pre-class quiz

Premise 1: $p \rightarrow q$

Premise 2: $p \vee r$

Premise 3: $p \vee \sim r$

Conclusion: q

1: $p \rightarrow q$

2: $p \vee r$

3: $p \vee \sim r$

4: $(p \vee r) \wedge (p \vee \sim r)$ from 2 and 3

5: $p \vee (r \wedge \sim r)$ from 4 by the Distributive law

6: $p \vee \mathbf{c}$ from 5 by the Negation law

7: p from 6 by the Identity law

Conclusion: q from 1 and 7 by Modus Ponens

Proving an argument is valid (by checking if the argument is invalid):**Premise 1:** $w \rightarrow b$ **Premise 2:** $d \rightarrow \sim b$ **Conclusion:** $\sim d \vee \sim w$

Activity 5: Try to find truth values for the statement variables that make all the premises true but the conclusion false (and hence demonstrate that the argument is invalid).

Conclusion: $\sim d \vee \sim w$ For this to be false we require $\sim d$ to be false and $\sim w$ to be false. Thus we have d true and w true.

Premise 1: $w \rightarrow b$ For this to be true, given that we already know that w is true, we require b to be true. Thus we have b true.

Premise 2: $d \rightarrow \sim b$ For this to be true, given that we already know that d is true, we require $\sim b$ to be true. Thus we have b false.

This is a contradiction, so it is **not possible** to find truth values for the variables w, b, d that make all the premises true and the conclusion false, so the argument is valid.

Q4 from the L5 pre-class quiz**Premise 1:** $p \rightarrow q$ **Premise 2:** $q \rightarrow r$ **Premise 3:** $\sim p \vee \sim q$ **Conclusion:** r

Find truth values for the statement variables that make all the premises true but the conclusion false (and hence demonstrate that the argument is invalid).

Conclusion: r For this to be false we require r false.

Premise 2: $q \rightarrow r$ For this to be true, given that we already know that r is false, we require q false.

Premise 1: $p \rightarrow q$ For this to be true, given that we already know that q is false, we require p false.

Premise 3: $\sim p \vee \sim q$ We already know that p is false and q is false. Hence this premise is true.

So with p false, q false and r false, the premises are all true and the conclusion is false, so these truth values demonstrate that the argument is invalid.